A STUDY ON THE WIDE-SENSE STATIONARITY OF THE UNDERWATER ACOUSTIC CHANNEL FOR COHERENT COMMUNICATION SYSTEMS

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Abstract: *In this paper, we test the Wide-Sense Stationarity (WSS) of the estimation error achieved by an exponentially weighted recursive least squares (RLS) algorithm, used to estimate the channel impulse response. We study the stationarity of this process because it is closely tied to the bit error rate performance of adaptive equalizers used for symbol estimation. The predictability of this error process is related to the predictability of the quality of the communications channel. Most adaptive estimators are based upon estimated channel statistics which assume that the process is wide sense stationary. We analyze the data set collected in the SPACE08 experiment, during which the environmental conditions varied significantly in time, potentially causing a non-stationary behavior of the channel impulse response. The considered stationarity test is based on an analysis of the frequency-time properties of the signal and does not assume any model on how the data has been generated. This study aims at investigating the possibility of developing a predictor of the communications performance in order to trigger adaptive algorithms. Moreover, these results are useful to estimate the channel statistics for coherent communications systems and develop better channel models for shallow water.*

Keywords: *Stationarity test, underwater acoustic channel, adaptive algorithms, underwater communications*

1. INTRODUCTION

In this paper, we study the WSS of the prediction error of an exponentially weighted Recursive Least Squares (RLS) algorithm, responsible for estimating the channel impulse response.

We test the WSS property of the prediction error process, because we are interested in analyzing the performance of data adaptive estimators able to predict the state of the communication system, given some observations. Nevertheless, those estimators are optimized for some statistics of the process, thus assuming that the WSS is verified at least over the time interval of the estimation and prediction. These predictors might play an important role in triggering adaptive algorithms , in order to improve the efficiency of the communication systems at the cost of a slightly more complicated system.

We focus on the prediction error of a channel estimator, because it is strongly related to the performance of an equalizer, which, in case of coherent communication systems, is always employed. Finally, the choice of the RLS adaptive algorithm is justified by its robustness to slow variations of the channel statistics, thus resulting in a suitable algorithm for our case of study.

In [1], we tested the WSS of the channel energy, which is a meaningful metric for non-coherent systems, and we observed the relationship between the environmental conditions and the statistics of the channel energy. Differently, here we focus on a metric, which is more representative of the performance of coherent receiving systems, and we will show how the statistics vary over time in case we consider the effect of phase changes.

We use the algorithm in [2] among other algorithms since it does not assume any model that generates the data, but analogously to the standard stationarity test, it studies the variations of the local spectra with respect to the global spectrum. In order to do so, the authors propose to generate a surrogate data set, which is a set of realizations of a stationarized version of the original data set. In this way we can compare the variations, between each local spectrum with respect to the global spectrum, observed in the surrogate data set, and those observed in the original data set.

This work is one of the first contributions in testing the wide-sense stationarity over particular time intervals of a metric representing the quality of the underwater channel. In the literature, we find some similar studies for wireless channels. In [3] a stationarity test and an experimental study have been presented for MIMO mobile wireless channels. Moreover, other tests of the WSS for the mobile wireless channel on experimental data sets have been published in [4], [5] and [6].

Unlike its wireless counterpart, the underwater acoustic channel might show some non-stationarity even if it is measured between static nodes, mainly due to the time-varying environmental conditions, such as the surface roughness. Indeed, especially in shallow water, which is the case we analyze here, the acoustic waves interact with the surface waves and depending on the surface conditions and time variability, this gives rise to non-stationarity of the received acoustic field.

We will show here the time scale in which the error process can be considered stationary. Even if we are studying a specific case in a specific environment, we expect that our conclusions are widely representative of the effects studies, and that the same results could be applied to scenarios with similar geometries.

2. DATA SET AND DESCRIPTION OF THE METRIC REPRESENTING THE CHANNEL QUALITY

In this section we introduce the data set called Surface Processes and Acoustic Communications Experiment (SPACE08) and we present the processing that we performed on the data. The reader can find a more detailed description of the data set in [1]. For the sake of completeness we will describe shortly its meaningful characteristics for this work.

The data set is a collection of recorded acoustic signals and environmental measurements. The experiments were performed in a location off the coast of Martha's Vineyard island during 18-27 October 2008. The scenario consists of one transmitter, six fixed receiving systems, and other instruments used to measure environmental conditions (i.e., surface waves, wind speed and direction). We consider the data received at systems S3 and S4, which were located at 200 m from the transmitter at Southeast and Southwest directions, respectively, and at systems S5 and S6 at 1000 m from the transmitter, aligned respectively with S3 and S4. The water depth was almost uniform in the whole experiment area and was about 15 m. This implies that the acoustic field was mostly affected by the surface waves, which are in turn dependent on the wind speed and direction. Figure 1 shows the changing wind conditions during the experiments. The acoustic signals are a repetition of a 4095 point binary m-sequence, transmitted at a symbol rate of 6.5 kbps and modulated at a central frequency of 11.5 kHz.

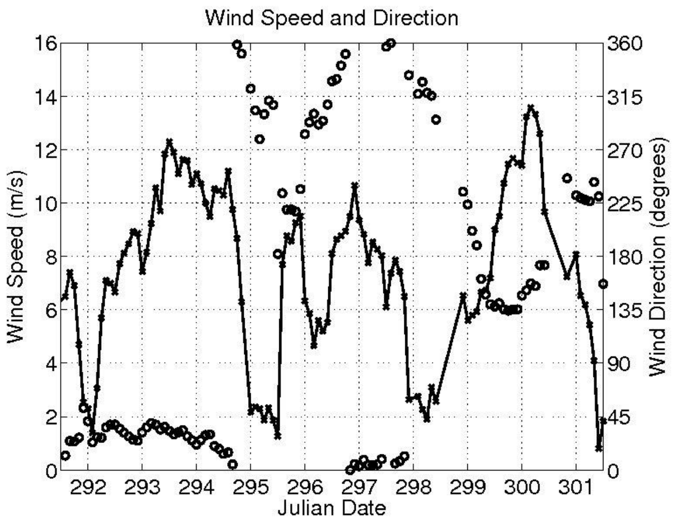

Fig.1: Wind speed and direction during SPACE08. The continue line is the wind speed while the circles represent the direction.

For each received signal, we run the exponential weighted RLS algorithm [7] to estimate the channel and compute the prediction error. The exponential weighted least square algorithm is known to minimize the cost function defined as

$$C(n) = \sum_{i=1}^{n} \lambda^{n-i} \left| e(i) \right|^2 . \tag{1}$$

The parameter λ is a memory factor, which indicates the weight of the prediction error at i time unites earlier and λ is defined in the following. We assume that the channel has a limited duration M and we let the column vector $\mathbf{w}(n)$ be the channel estimate at time n. We denote the transmitted symbols at times $n-M+1$ to n as $\mathbf{u}(n)$ and the received signal at time n as $r(n)$. The computed metric, which we call the prediction error, can be expressed as

$$e(n) = r(n) - \mathbf{w}(n-1)^{*}\mathbf{u}(n), \tag{2}$$

where the symbol * indicates conjugate transposed. Therefore, $e(n)$ is a measure of how accurately the present received value can be predicted using the previous channel estimate. This accuracy is related to the performance of the equalizer and therefore to the performance of the communication system. The parameters used in our implementation are $M = 8$ ms and $\lambda = 0.95$.

3. STATIONARITY TEST

In this section we briefly describe the stationarity test that we employed in this study. We consider the methodology proposed in [2], and refer the reader to that paper for a more detailed description. We are interested in testing the wide-sense stationarity of the process, which is verified when the first (mean) and the second moment (the correlation) of the process are independent of time.

The framework in [2] is suitable for our study, because it does not assume any model for the system that generated the data, and the test is based on the comparison of time-frequency features between stationarized versions of the data, called surrogates, and the original data set. We compute the local spectra by using the multitaper spectrogram defined as:

$$S_{e,K}(t,f) = \frac{1}{K}\sum_{1}^{K} S_e^{h_k}(t,f), \tag{3}$$

where $S_e^{h_k}(t,f)$ is the spectrogram of the process $e(t)$ computed with the k-th Hermite function, whose length we indicate as T_h. We obtain N local spectra.

Given that we are analyzing an experimental data set, we expect to observe always a variability of the local spectra with respect to the global spectrum (obtained by marginalization, i.e., $G_s(f) = E\left(S_{e,K}(t,f)\right)$), therefore we will compare this variability to the fluctuation between the local and global spectra in the surrogate data set. The surrogate data set is computed by multiplying the amplitude of the Fourier transform of the original time series by an independent identically distributed phase sequence and then applying the inverse Fourier transform. In this way the correlation function of the obtained process depends only on the interval between two sequences and not on the absolute times at which they were taken, i.e. the WSS property is satisfied. We calculate J realizations of the surrogate data set. We evaluate the distance between the local spectra and the global spectrum $G_s(f)$ as a combination of the Kullback-Leibler divergence defined as:

$$D_{KL}(S_{e,K}, G_s) = \int_{\Omega} \left(S_{e,K}(f) - G_s(f)\right) \log\frac{S_{e,K}(f)}{G_s(f)} df \tag{4}$$

and the log-spectral deviation expressed as:

$$D_{LDS}(S_{e,K},G_s)=\int_{\Omega}\left|\log\frac{S_{e,K}(f)}{G_s(f)}\right|df \quad . \tag{5}$$

The considered combination is:

$$D(S_{e,K},G_s)=D_{KL}(s_{e,K},g_s)\times\left(+D_{LSD}(S_{e,K},G_s)\right) \tag{6}$$

$s_{e,K}$ and g_s are respectively normalized versions of $S_{e,K}$ and G_s, i.e., they are obtained by dividing $S_{e,K}$ and G_s by their integral. We compute the N distances for each surrogate and for the experimental data, thus obtaining $J+1$ sets of N distances each. We then compute the variance of each set and we indicate as Θ_1 the variance of the N distances over the experimental data set, while we denote as Θ_0 the set of J variances of the distances over the surrogates. The authors in [2] showed that the elements of Θ_0 can be thought as drawn from a Gamma distribution $\Gamma(x;a.b)$, which depends on the positive parameters a and b. We estimate these parameters from set Θ_0. We choose a probability of failure of 5% and, from the cumulative distribution function, we determine the threshold (α) of the variance such that the probability that the variance is less than or equal to α is 95%. Then the test result is "stationary" if $\Theta_1 \le \alpha$ and "non-stationary" otherwise. We perform this test for different values of T_h and we consider the largest value of T_h such that the test result is "stationary" as the interval of stationarity. In the following section we will present the parameters used and show the results.

4. RESULTS

In this section we show the results and provide some qualitative interpretation of the results by comparing the environmental and the acoustic data.

In the implementation of the test we use $J=50$, $\alpha=0.05$ and T_h varies from 8 s to 80 s. We are interested in verifying the stationarity over time intervals of the order of tens of seconds, because it opens the possibility to use stochastic predictors, or data adaptive estimators, which assume the stationarity property for upper layer protocols. If the process is found to be stationary over an interval of the orders of several seconds, we might be able to estimate the statistics of the process in order to optimize the performance of the communications and in order to take advantage of this statistics in protocols that take place over time intervals of tens of seconds (considering also the feedback) such as hybrid automatic repeat request techniques, medium access control and routing protocols. The time series is 160 s long (indeed every signal lasts less than one minute and is repeated three times), thus in order to compute a local spectrum different from the global spectrum, the maximum duration of the interval over which we test the stationarity is 80 s long.

We run the test for each receiving system and we observe that in certain periods the test gives at least a non stationarity result in the time series of almost three minutes. Figures 2 and 3 represent the results of the stationarity test respectively for systems S3 and S5. The y-axis indicates when the test detects a non stationarity interval T_h (-1), and when it does not (1), the x-axis represents the Julian date and the hour of the measurements (we recall that each measurement was taken every two hours). For reference, Julian date 292 corresponds to October 18, 2008.

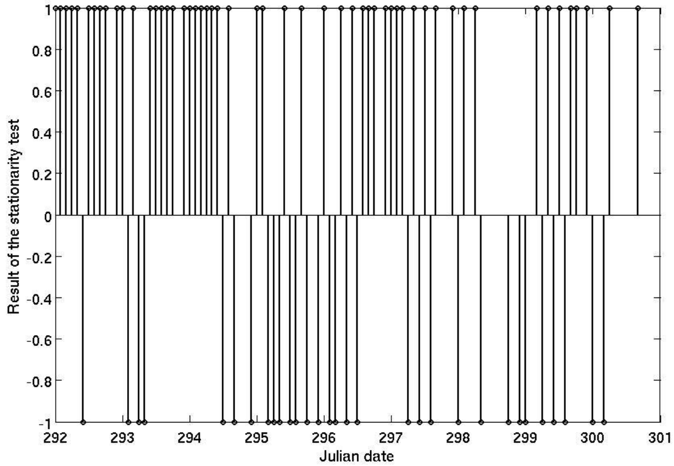

Fig.2: Result of the stationarity test for the SPACE08 data set and system S3 at 200 m from the transmitter at Southeast direction.

It can be noticed that system S5 is characterized by having more non stationarity detections than system S3, which can be explained by the fact that the variability of the channel statistics observed at system S5 is greater than that for S3, due to the larger number of interactions between the acoustic wave and the time-varying surface.

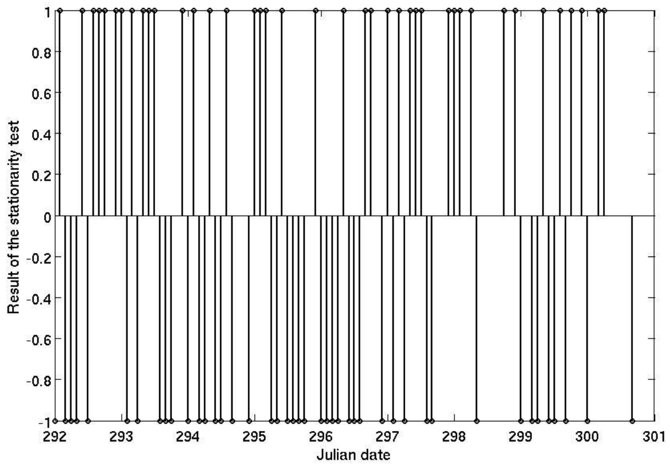

Fig.3: Result of the stationarity test for the SPACE08 data set and system S5at 1000 m from the transmitter and at Southeast direction.

Moreover we can observe that during the periods between Julian dates 295 and 296 and between 298 and 300 both systems show non stationarity conditions. This might be explained by the changes conditions of the wind speed in those periods.

We evaluate the probability that the stationarity interval is greater than a given value. We choose this metric, which we call Complementary Cumulative Distribution Function (CCDF), because it is representative of the probability that the process is stationary for a longer interval.

Figure 4 shows the CCDF for all the receiving systems. We notice that systems S3 and S4 are characterized by longer stationarity intervals than systems S5 and S6. This again can be explained by the larger number of interactions between the acoustic signal and the surface waves, thus resulting in a more variable statistics of the observed metric. Indeed the observed metric is sensitive to the phase changes due to the superposition of constructive and destructive interference resulting from the scattering generated by the interaction with the surface waves. A future step is to correlate the spectrum (computed over the stationarity interval) of the observed metric with the spectrum of the surface waves.

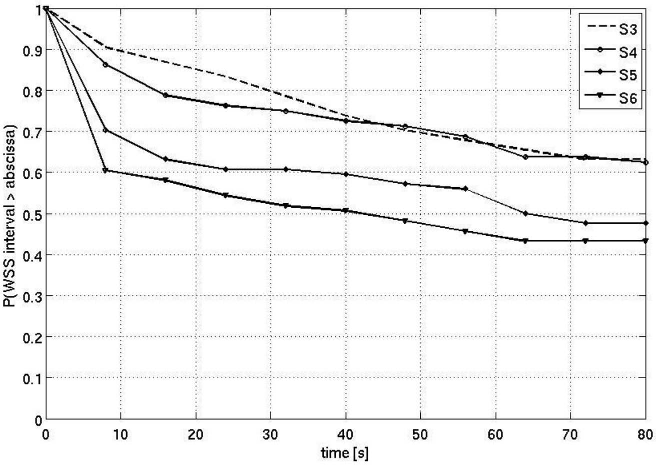

Fig.4: Complementary cumulative distribution function of the duration of the stationarity interval for all the receiving systems.

To conclude, we can state that data adaptive estimators might not perform optimally in longer links, due to the lack of stationarity of the observed process, although it remains to be better understood to what extent the communication performance degrades when the stationarity is not verified.

Nevertheless, these results give evidence that, in a shallow water scenario, shorter links seem to be more suitable for systems that use predictors and adaptive algorithms.

5. CONCLUSION

In this paper we tested the stationarity of the error process which is a channel dependent metric and is also related to communications performance. We used the methodology described in [2] and considered the SPACE08 data set, which is characterized by time varying environmental conditions, and is particularly suitable for these kinds of statistical evaluations. The results show that shorter links are characterized by longer stationarity intervals than those at greater distances.

As part of our future work, we want to analyze the performance of stochastic predictors, in case of stationary channel conditions, and to evaluate the improvement of the system performance when adaptive algorithms are triggered by those predictors.

6. ACKNOWLEDGMENT

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